Related topics
Longitudinal waves, Huygens’ principle, Interference, Fraunhofer and Fresnel diffraction, Fresnel’s zone construction; Poisson’ spot, Babinet’ theorem, Bessel function.

Principle
An ultrasonic plane wave is subjected to diffraction by a pin-hole obstacle and a complementary circular obstacle. The intensity distribution of the diffracted and interfering partial waves are automatically recorded using a motor-driven, swivel ultrasound detector and a PC.

Equipment
Goniometer with reflecting mirror 13903.00 1
Power supply for goniometer 13903.99 1
Ultrasonic unit 13900.00 1
Power supply f. ultrasonic unit, 5 VDC, 12 W 13900.99 1
Ultrasonic transmitter on stem 13901.00 1
Ultrasonic receiver on stem 13902.00 1
Object holder for ultrasonic 13904.00 1
Pin hole and circular obstacle for ultrasonic 13906.00 1
RS 232 data cable 14602.00 1
Measuring tape, l = 2m 09936.00 1
Screened cable, BNC, l= 75 cm 07542.11 1
Adapter, BNC-socket/4 mm plug pair 07542.27 1
Measure Software Goniometer 14523.61 1
PC, Windows® 95 or higher

Tasks
1. Determine the angular distribution of an ultrasonic wave diffracted by a pin-hole and circular obstacle.
2. Compare the angular positions of the minimum intensities with the theoretical values.

Set-up and Procedure
Set up the experiment as shown in Fig. 1. Exact adjustment of the experimental set-up is important!

Adjustment of the goniometer:
– Use the adjusting screws at the back of the mirror and under its stem to set the mirror by eye to a vertical position and align it to the zero line of the goniometer table.
– Slide the transmitter tightly against the mirror and align it to the height of the centre of the mirror.
– Slide the transmitter back to fit the 16 cm long adjusting rod in the hole in the centre of the mirror. The rod must point directly to the middle of the transmitter. Should this not be the case, again use the adjusting screws to readjust the mirror. Remove the rod so that the transmitter can be brought to the focal point of the mirror. The distance from the centre of the mirror must be exactly 15.5 cm (measuring tape).
– To adjust the height of the receiver, turn it with its swing arm as near as possible to the mirror. It might be necessary here to first unlock the transport stop (to do this, pull the yellow screw of the swing arm beneath the goniometer table). Bring the receiver to the same height as the transmitter.
– Set the receiver swing arm to zero. The axis of the receiver must correspond with the goniometer zero line.
– When adjustment is properly made, the axes of the mirror, transmitter and receiver are on a common line which must lie exactly above the zero line of the goniometer table.

Fig. 1: Experimental set-up.
Connection of instruments
Connect the transmitter to the diode socket of the ultrasonic unit that is marked TR1, and operate it in "Con" continuous mode. Connect the receiver to the left BNC socket (prior to the amplifier). Further, use the BNC cable to connect the analog output of the ultrasonic unit to the input of the control unit (pay attention to the polarity of the adapter), and the latter unit to the PC by means of the RS 232 data cable. For control of the goniometer, connect the socket underneath the goniometer plate with the control unit.
With the “Cal” key of the control unit pressed (release of the motor drive) position the swing arm at 0°. Following this, deactivate the “Cal” function.
To ensure proportionality between the input signal of the receiver and its analog output signal, avoid operating the ultrasonic unit amplifier in the saturation range. Should this occur and the “OVL” diode light up, reduce either the transmitter amplitude or the input amplification of the receiver. It is purposeful here to adjust the amplification at the zero position of the receiver so that the “OVL” diode just no longer lights up.

Experiment with the pinhole diaphragm
- Fit the centering pin of the object holder into the centering socket of the goniometer table, then adjust the object holder to be in alignment with the 90° line of the goniometer table. The feet of the object holder should hereby point to the mirror.
- Carefully push the diaphragm in the guide grooves of the object holder, then align it to be exactly centrally symmetric.
- To avoid interfering sound reflections between object holder and mirror, use the carrier foam as wave absorbent by placing it tightly against the holder, with its opening symmetrically towards the pinhole diaphragm.
- Use the software to set the range of swing of the receiver to ±50°.
- Bring the receiver to the middle of the swing arm.

Experiment with the circular diaphragm
- The circular diaphragm is to be held by one of the feet of the object holder. Mount the circular diaphragm vertically on the foot and position it exactly at the centre of the goniometer table. To be able to do this, first remove the centering socket from the table.
- Use the software to set the range of swing of the receiver to ±15°.
- Bring the receiver to the middle of the swing arm.

Note:
Faulty intensity modulation may occur in spectra as a result of interference in the measurement field. To keep such interference as small as possible, do not carry out experiments in too narrow rooms or in the direct vicinity of reflecting surfaces (walls, cupboards etc.). It is recommended that the measuring and supply instruments be installed behind the mirror if possible. Further to this, the person carrying out the experiment should not stand too close to the measurement field. Should asymmetries occur in the spectral intensities, these can as a rule be avoided by slightly turning the object holder or diaphragm around the 90° line on the goniometer table.

Theory and Evaluation
When a wave hits a pinhole diaphragm then, acc. to Huygens’ principle, spherical waves emanate from each point of the slit opening. The individual partial waves interfere with each other behind the obstacle. According to their phase position, they intensify each other in certain directions, or extinguish each other.
In such cases, the following is valid for the angular distribution of the sound pressure $p(\phi)$:

$$p(\phi) \propto p(\phi = 0) - \frac{2\pi}{\lambda} R \sin \varphi \sin \left( \frac{2\pi}{\lambda} R \sin \varphi \right)$$

(1)

(where $R$ = radius of the pinhole diaphragm, $\lambda$ = wavelength and $J_1$ = 1st order Bessel function)
This distribution function resembles that for a slit, but has the 1st order Bessel function $J_1$ instead of the sine. This has zero positions at:

$$\sin \varphi_1 = 0.610 \frac{\lambda}{R}; \sin \varphi_2 = 1.116 \frac{\lambda}{R};$$

$$\sin \varphi_3 = 1.619 \frac{\lambda}{R}; \sin \varphi_4 = 2.120 \frac{\lambda}{R};$$

(2)

Fig. 2 shows the interference pattern behind a pinhole diaphragm of radius $R = 2.5$ cm. Table 1 contains the corresponding evaluation of this. As the central maximum does not generally lie exactly at 0°, it is preferable to determine the angular distance $2 \varphi$ from the measurement curve using two minima lying symmetrically to the zero line.

Fig. 2: Interference pattern of ultrasonic waves diffracted at a pinhole diaphragm.
Table 1: Evaluation of the minima of the interference curve in Fig. 2.

<table>
<thead>
<tr>
<th>n</th>
<th>$2\varphi/\pi$</th>
<th>$\lambda$/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.0</td>
<td>8.52</td>
</tr>
<tr>
<td>2</td>
<td>46.0</td>
<td>8.75</td>
</tr>
<tr>
<td>3</td>
<td>68.9</td>
<td>8.74</td>
</tr>
<tr>
<td>4</td>
<td>94.6</td>
<td>8.67</td>
</tr>
</tbody>
</table>

The mean value of the wavelength values listed in Table 1 is: $\lambda = (0.867 \pm 0.01)$ cm. The transmitter operates at a frequency of $f = 40$ kHz. From $c = f \cdot \lambda$ ($c = 343.4$ ms$^{-1}$ at $T = 20^\circ$C) it follows that, in complete agreement with the experiment, $\lambda = 0.858$ cm. When the pinhole diaphragm is replaced by a complementary circular disc, then Babinet’s theorem says that an intensity maximum will always be present in the area of the geometric shadow behind the diaphragm but that, compared with the central maximum of the complementary pinhole diaphragm, this intensity maximum will be symmetrically intersected by two minima (see Fig. 3).

In the area outside of the geometric shadow, the manifestation of the diffraction is comparable to that obtained using a pin-hole diaphragm.

Fig. 3: Interference pattern of ultrasonic waves diffracted at a complementary circular disc.
Diffraction of ultrasonic waves at a pin hole and a circular obstacle